Emergent Cosmology, Inflation and Dark Energy from Spontaneous Breaking of Scale Invariance

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Abstract A new class of gravity-matter models defined in terms of two independent non-Riemannian volume forms (alternative generally covariant integration measure densities) on the space-time manifold are studied in some detail. These models involve an additional R^2 (square of the scalar curvature) term as well as scalar matter field potentials of appropriate form so that the pertinent action is invariant under global Weyl-scale symmetry. Scale invariance is spontaneously broken upon integration of the equations of motion. After performing transition to the physical Einstein frame we obtain: (i) An effective potential for the scalar field with two flat regions which allows for a unified description of both early universe inflation as well as of present dark energy epoch; (ii) For a definite parameter range the model possesses a non-singular "emergent universe" solution which describes an initial phase of evolution that precedes the inflationary phase.

Keywords modified gravity theories, non-Riemannian volume forms, global Weyl-scale symmetry spontaneous breakdown, flat regions of scalar potential, non-singular origin of the universe

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1 Introduction

Modern cosmology has been formulated in an attractive framework where many aspects of the observable universe can be incorporated. In this "standard cosmological" framework, the early universe (cf. the books [1] and references therein) starts with a period of exponential expansion called "inflation". In the inflationary period also primordial density perturbations are generated (Ref.[2] and references

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therein). The "inflation" is followed by particle creation, where the observed matter and radiation were generated [1], and finally the evolution arrives to a present phase of slowly accelerating universe [3,4]. In this standard model, however, at least two fundamental questions remain unanswered:

– The early inflation, although solving many cosmological puzzles, like the horizon and flatness problems, cannot address the initial singularity problem;
– There is no explanation for the existence of two periods of exponential expansion with such wildly different scales – the inflationary phase and the present phase of slowly accelerated expansion of the universe.

The best known mechanism for generating a period of accelerated expansion is through the presence of some vacuum energy. In the context of a scalar field theory, vacuum energy density appears naturally when the scalar field acquires an effective potential $U_{\rm eff}$ which has flat regions so that the scalar field can "slowly roll" [5,6] and its kinetic energy can be neglected resulting in an energy-momentum tensor $T_{\mu\nu} \simeq g_{\mu\nu} U_{\rm eff}$.

The possibility of continuously connecting an inflationary phase to a slowly accelerating universe through the evolution of a single scalar field – the *quintessential inflation scenario* – has been first studied in Ref.[7]. Also, $F(R)$ models can yield both an early time inflationary epoch and a late time de Sitter phase with vastly different values of effective vacuum energies [8]. For a recent proposal of a quintessential inflation mechanism based on the k-essence [9] framework, see Ref.[10]. For another recent approach to quintessential inflation based on the "variable gravity" model [11] and for extensive list of references to earlier work on the topic, see Ref.[12].

In the present paper we will study a unified scenario where both an inflation and a slowly accelerated phase for the universe can appear naturally from the existence of two flat regions in the effective scalar field potential which we derive systematically from a Lagrangian action principle. Namely, we start with a new kind of globally Weyl-scale invariant gravity-matter action within the first-order (Palatini) approach formulated in terms of two different non-Riemannian volume forms (integration measures) [13]. In this new theory there is a single scalar field with kinetic terms coupled to both non-Riemannian measures, and in addition to the scalar curvature term R also an R^2 term is included (which is similarly allowed by global Weyl-scale invariance). Scale invariance is spontaneously broken upon solving part of the corresponding equations of motion due to the appearance of two arbitrary dimensionful integration constants. We find in the physical Einstein frame an effective k-essence [9] type of theory, where the effective scalar field potential has *two flat regions* corresponding to the two accelerating phases of the universe – the inflationary early universe and the present late universe.

In addition, within the flat region corresponding to the early universe we also obtain another phase that precedes the inflation and provides for a non-singular origin of the universe. It is of an "emergent universe" type [14], *i.e.*, the universe starts as a static Einstein universe, the scalar field rolls with a constant speed through a flat region and there is a domain in the parameter space of the theory where such non-singular solution exists and is stable. To this end let us recall that the concept of "emergent universe" solves one of the principal puzzles in cosmology – the problem of initial singularity [15] including avoiding the singularity theorems for scalar field-driven inflationary cosmology [16].

Let us briefly recall the origin of current approach. The main idea comes from Refs.[17]-[19] (see also [20,21]), where some of us have proposed a new class of gravity-matter theories based on the idea that the action integral may contain a new metric-independent generally-covariant integration measure density, *i.e.*, an alternative non-Riemannian volume form on the space-time manifold defined in terms of an auxiliary antisymmetric gauge field of maximal rank. The originally proposed modified-measure gravity-matter theories [17]-[21] contained two terms in the pertinent Lagrangian action – one with a non-Riemannian integration measure and a second one with the standard Riemannian integration measure (in terms of the square-root of the determinant of the Riemannian space-time metric). An important feature was the requirement for global Weyl-scale invariance which subsequently underwent dynamical spontaneous breaking [17]. The second action term with the standard Riemannian integration measure might also contain a Weyl-scale symmetry preserving R^2-term [19].

The latter formalism yields various new interesting results in all types of known generally covariant theories:

- (i) $D = 4$-dimensional models of gravity and matter fields containing the new measure of integration appear to be promising candidates for resolution of the dark energy and dark matter problems, the fifth force problem, and a natural mechanism for spontaneous breakdown of global Weyl-scale symmetry [17]-[21].
- (ii) Study of reparametrization invariant theories of extended objects (strings and branes) based on employing of a modified non-Riemannian world-sheet/world-volume integration measure [22] leads to dynamically induced variable string/brane tension and to string models of non-abelian confinement.
- (iii) Study in Refs.[23] of modified supergravity models with an alternative non-Riemannian volume form on the space-time manifold produces some outstanding new features: (a) This new formalism applied to minimal $N = 1$ supergravity naturally triggers the appearance of a dynamically generated cosmological constant as an arbitrary integration constant, which signifies a new explicit mechanism of spontaneous (dynamical) breaking of supersymmetry; (b) Applying the same formalism to anti-de Sitter supergravity allows us to appropriately choose the above mentioned arbitrary integration constant so as to obtain simultaneously a very small effective observable cosmological constant as well as a very large physical gravitino mass.

The plan of the present paper is as follows. In the next Section 2 we describe in some detail the general formalism for the new class of gravity-matter systems defined in terms of two independent non-Riemannian integration measures. In Section 3 we describe the properties of the two flat regions in the Einstein-frame effective scalar potential corresponding to the evolution of the early and late universe, respectively. In Section 4 we derive a non-singular "emergent universe" solution of the new gravity-matter system. We conclude in Section 5 with some discussions.

2 Gravity-Matter Formalism With Two Independent Non-Riemannian Volume-Forms

We shall consider the following non-standard gravity-matter system with an action of the general form involving two independent non-Riemannian integration

measure densities generalizing the model studied in [13] (for simplicity we will use units where the Newton constant is taken as $G_{\text{Newton}} = 1/16\pi$):

$$S = \int d^4x\, \Phi_1(A)\Big[R + L^{(1)}\Big] + \int d^4x\, \Phi_2(B)\Big[L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}}\Big] . \qquad (1)$$

Here the following notations are used:

- $\Phi_1(A)$ and $\Phi_2(B)$ are two independent non-Riemannian volume-forms, *i.e.*, generally covariant integration measure densities on the underlying space-time manifold:

$$\Phi_1(A) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu A_{\nu\kappa\lambda} \quad , \quad \Phi_2(B) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu B_{\nu\kappa\lambda} , \qquad (2)$$

  defined in terms of field-strengths of two auxiliary 3-index antisymmetric tensor gauge fields[1]. $\Phi_{1,2}$ take over the role of the standard Riemannian integration measure density $\sqrt{-g} \equiv \sqrt{-\det\|g_{\mu\nu}\|}$ in terms of the space-time metric $g_{\mu\nu}$.
- $R = g^{\mu\nu}R_{\mu\nu}(\Gamma)$ and $R_{\mu\nu}(\Gamma)$ are the scalar curvature and the Ricci tensor in the first-order (Palatini) formalism, where the affine connection $\Gamma^\mu_{\nu\lambda}$ is a *priori* independent of the metric $g_{\mu\nu}$. Note that in the second action term we have added a R^2 gravity term (again in the Palatini form). Let us recall that $R + R^2$ gravity within the second order formalism (which was also the first inflationary model) was originally proposed in Ref.[24].
- $L^{(1,2)}$ denote two different Lagrangians of a single scalar matter field of the form (similar to the choice in Ref.[17]):

$$L^{(1)} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi) \quad , \quad V(\varphi) = f_1\exp\{-\alpha\varphi\} , \qquad (3)$$

$$L^{(2)} = -\frac{b}{2}e^{-\alpha\varphi}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + U(\varphi) \quad , \quad U(\varphi) = f_2\exp\{-2\alpha\varphi\} , \qquad (4)$$

  where α, f_1, f_2 are dimensionful positive parameter, whereas b is a dimensionless one.
- $\Phi(H)$ indicates the dual field strength of a third auxiliary 3-index antisymmetric tensor gauge field:

$$\Phi(H) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu H_{\nu\kappa\lambda} , \qquad (5)$$

  whose presence is crucial for non-triviality of the model.

The scalar potentials have been chosen in such a way that the original action (1) is invariant under global Weyl-scale transformations:

$$g_{\mu\nu} \to \lambda g_{\mu\nu} \;\; , \;\; \varphi \to \varphi + \frac{1}{\alpha}\ln\lambda \;\; ,$$

$$A_{\mu\nu\kappa} \to \lambda A_{\mu\nu\kappa} \;\; , \;\; B_{\mu\nu\kappa} \to \lambda^2 B_{\mu\nu\kappa} \;\; , \;\; H_{\mu\nu\kappa} \to H_{\mu\nu\kappa} . \qquad (6)$$

For the same reason we have multiplied by an appropriate exponential factor the scalar kinetic term in $L^{(2)}$ and also R and R^2 couple to the two different modified measures because of the different scalings of the latter.

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[1] In D space-time dimensions one can always represent a maximal rank antisymmetric gauge field $A_{\mu_1\ldots\mu_{D-1}}$ in terms of D auxiliary scalar fields ϕ^i ($i = 1,\ldots,D$) in the form: $A_{\mu_1\ldots\mu_{D-1}} = \frac{1}{D}\varepsilon_{ii_1\ldots i_{D-1}}\phi^i\partial_{\mu_1}\phi^{i_1}\ldots\partial_{\mu_{D-1}}\phi^{i_{D-1}}$, so that its (dual) field-strength $\Phi(A) = \frac{1}{D!}\varepsilon_{i_1\ldots i_D}\varepsilon^{\mu_1\ldots\mu_D}\partial_{\mu_1}\phi^{i_1}\ldots\partial_{\mu_D}\phi^{i_D}$.

The equations of motion resulting from the action (1) are as follows. Variation of (1) w.r.t. affine connection $\Gamma^{\mu}_{\nu\lambda}$:

$$\int d^4 x \sqrt{-g} g^{\mu\nu} \left(\frac{\Phi_1}{\sqrt{-g}} + 2\epsilon \frac{\Phi_2}{\sqrt{-g}} R \right) \left(\nabla_\kappa \delta \Gamma^\kappa_{\mu\nu} - \nabla_\mu \delta \Gamma^\kappa_{\kappa\nu} \right) = 0 \qquad (7)$$

shows, following the analogous derivation in the Ref.[17], that $\Gamma^{\mu}_{\nu\lambda}$ becomes a Levi-Civita connection:

$$\Gamma^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{\nu\lambda}(\bar{g}) = \frac{1}{2} \bar{g}^{\mu\kappa} \left(\partial_\nu \bar{g}_{\lambda\kappa} + \partial_\lambda \bar{g}_{\nu\kappa} - \partial_\kappa \bar{g}_{\nu\lambda} \right) , \qquad (8)$$

w.r.t. to the Weyl-rescaled metric $\bar{g}_{\mu\nu}$:

$$\bar{g}_{\mu\nu} = (\chi_1 + 2\epsilon\chi_2 R) g_{\mu\nu} , \quad \chi_1 \equiv \frac{\Phi_1(A)}{\sqrt{-g}} , \quad \chi_2 \equiv \frac{\Phi_2(B)}{\sqrt{-g}} . \qquad (9)$$

Variation of the action (1) w.r.t. auxiliary tensor gauge fields $A_{\mu\nu\lambda}$, $B_{\mu\nu\lambda}$ and $H_{\mu\nu\lambda}$ yields the equations:

$$\partial_\mu \left[R + L^{(1)} \right] = 0 \quad , \quad \partial_\mu \left[L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}} \right] = 0 \quad , \quad \partial_\mu \left(\frac{\Phi_2(B)}{\sqrt{-g}} \right) = 0 , \quad (10)$$

whose solutions read:

$$\frac{\Phi_2(B)}{\sqrt{-g}} \equiv \chi_2 = \text{const} , \quad R + L^{(1)} = -M_1 = \text{const} , \quad L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = \text{const} .$$
$$(11)$$

Here M_1 and M_2 are arbitrary dimensionful and χ_2 arbitrary dimensionless integration constants. The appearance of M_1, M_2 signifies *dynamical spontaneous breakdown* of global Weyl-scale invariance under (6) due to the scale non-invariant solutions (second and third ones) in (11), whereas the first one with χ_2 preserves scale invariance.

Varying (1) w.r.t. $g_{\mu\nu}$ and using relations (11) we have:

$$\chi_1 \left[R_{\mu\nu} + \frac{1}{2} \left(g_{\mu\nu} L^{(1)} - T^{(1)}_{\mu\nu} \right) \right] - \frac{1}{2} \chi_2 \left[T^{(2)}_{\mu\nu} + g_{\mu\nu} \left(\epsilon R^2 + M_2 \right) - 2R R_{\mu\nu} \right] = 0 , \quad (12)$$

where χ_1 and χ_2 are defined in (9), and $T^{(1,2)}_{\mu\nu}$ are the energy-momentum tensors of the scalar field Lagrangians with the standard definitions:

$$T^{(1,2)}_{\mu\nu} = g_{\mu\nu} L^{(1,2)} - 2 \frac{\partial}{\partial g^{\mu\nu}} L^{(1,2)} . \qquad (13)$$

Taking the trace of Eqs.(12) and using again second relation (11) we solve for the scale factor χ_1:

$$\chi_1 = 2\chi_2 \frac{T^{(2)}/4 + M_2}{L^{(1)} - T^{(1)}/2 - M_1} , \qquad (14)$$

where $T^{(1,2)} = g^{\mu\nu} T^{(1,2)}_{\mu\nu}$.

Using second relation (11) Eqs.(12) can be put in the Einstein-like form:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{2} g_{\mu\nu} \left(L^{(1)} + M_1 \right) + \frac{1}{2\Omega} \left(T^{(1)}_{\mu\nu} - g_{\mu\nu} L^{(1)} \right)$$
$$+ \frac{\chi_2}{2\chi_1 \Omega} \left[T^{(2)}_{\mu\nu} + g_{\mu\nu} \left(M_2 + \epsilon (L^{(1)} + M_1)^2 \right) \right] , \qquad (15)$$

where:

$$\Omega = 1 - \frac{\chi_2}{\chi_1} 2\epsilon \left(L^{(1)} + M_1 \right) . \tag{16}$$

Let us note that (9), upon taking into account second relation (11) and (16), can be written as:

$$\bar{g}_{\mu\nu} = \chi_1 \Omega \, g_{\mu\nu} . \tag{17}$$

Now, we can bring Eqs.(15) into the standard form of Einstein equations for the rescaled metric $\bar{g}_{\mu\nu}$ (17), *i.e.*, the Einstein-frame equations:

$$R_{\mu\nu}(\bar{g}) - \frac{1}{2}\bar{g}_{\mu\nu} R(\bar{g}) = \frac{1}{2} T_{\mu\nu}^{\text{eff}} \tag{18}$$

with energy-momentum tensor corresponding (according to (13)) to the following effective (Einstein-frame) scalar field Lagrangian:

$$L_{\text{eff}} = \frac{1}{\chi_1 \Omega} \left\{ L^{(1)} + M_1 + \frac{\chi_2}{\chi_1 \Omega} \left[L^{(2)} + M_1 + \epsilon (L^{(1)} + M_1)^2 \right] \right\} . \tag{19}$$

In order to explicitly write L_{eff} in terms of the Einstein-frame metric $\bar{g}_{\mu\nu}$ (17) we use the short-hand notation for the scalar kinetic term:

$$X \equiv -\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi \tag{20}$$

and represent $L^{(1,2)}$ in the form:

$$L^{(1)} = \chi_1 \Omega \, X - V \quad , \quad L^{(2)} = \chi_1 \Omega \, be^{-\alpha\varphi} X + U , \tag{21}$$

with V and U as in (3)-(4).

From Eqs.(14) and (16), taking into account (21), we find:

$$\frac{1}{\chi_1 \Omega} = \frac{(V - M_1)}{2\chi_2 \left[U + M_2 + \epsilon(V - M_1)^2 \right]} \left[1 - \chi_2 \left(\frac{be^{-\alpha\varphi}}{V - M_1} - 2\epsilon \right) X \right] . \tag{22}$$

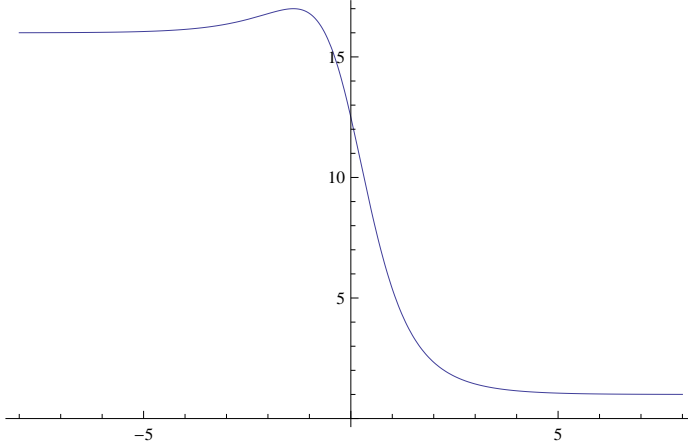Upon substituting expression (22) into (19) we arrive at the explicit form for the Einstein-frame scalar Lagrangian:

$$L_{\text{eff}} = A(\varphi)X + B(\varphi)X^2 - U_{\text{eff}}(\varphi) , \tag{23}$$

where:

$$A(\varphi) \equiv 1 + \left[\frac{1}{2}be^{-\alpha\varphi} - \epsilon(V - M_1) \right] \frac{V - M_1}{U + M_2 + \epsilon(V - M_1)^2}$$

$$= 1 + \left[\frac{1}{2}be^{-\alpha\varphi} - \epsilon\left(f_1 e^{-\alpha\varphi} - M_1 \right) \right] \frac{f_1 e^{-\alpha\varphi} - M_1}{f_2 e^{-2\alpha\varphi} + M_2 + \epsilon(f_1 e^{-\alpha\varphi} - M_1)^2} , \tag{24}$$

and

$$B(\varphi) \equiv \chi_2 \frac{\epsilon \left[U + M_2 + (V - M_1)be^{-\alpha\varphi} \right] - \frac{1}{4}b^2 e^{-2\alpha\varphi}}{U + M_2 + \epsilon(V - M_1)^2}$$

$$= \chi_2 \frac{\epsilon \left[f_2 e^{-2\alpha\varphi} + M_2 + (f_1 e^{-\alpha\varphi} - M_1)be^{-\alpha\varphi} \right] - \frac{1}{4}b^2 e^{-2\alpha\varphi}}{f_2 e^{-2\alpha\varphi} + M_2 + \epsilon(f_1 e^{-\alpha\varphi} - M_1)^2} , \tag{25}$$

Fig. 1 Shape of the effective scalar potential $U_{\text{eff}}(\varphi)$ (26) for $M_1 < 0$.

whereas the effective scalar field potential reads:

$$U_{\text{eff}}(\varphi) \equiv \frac{(V - M_1)^2}{4\chi_2 \left[U + M_2 + \epsilon(V - M_1)^2 \right]} = \frac{\left(f_1 e^{-\alpha\varphi} - M_1 \right)^2}{4\chi_2 \left[f_2 e^{-2\alpha\varphi} + M_2 + \epsilon(f_1 e^{-\alpha\varphi} - M_1)^2 \right]} ,$$
(26)

where the explicit form of V and U (3)-(4) are inserted.

Let us recall that the dimensionless integration constant χ_2 is the ratio of the original second non-Riemannian integration measure to the standard Riemannian one (9).

3 Flat Regions of the Effective Scalar Potential

Depending on the sign of the integration constant M_1 we obtain two types of shapes for the effective scalar potential $U_{\text{eff}}(\varphi)$ (26) depicted on Fig.1. and Fig.2.

The crucial feature of $U_{\text{eff}}(\varphi)$ is the presence of two very large flat regions – for negative and positive values of the scalar field φ. For large negative values of φ we have for the effective potential and the coefficient functions in the Einstein-frame scalar Lagrangian (23)-(26):
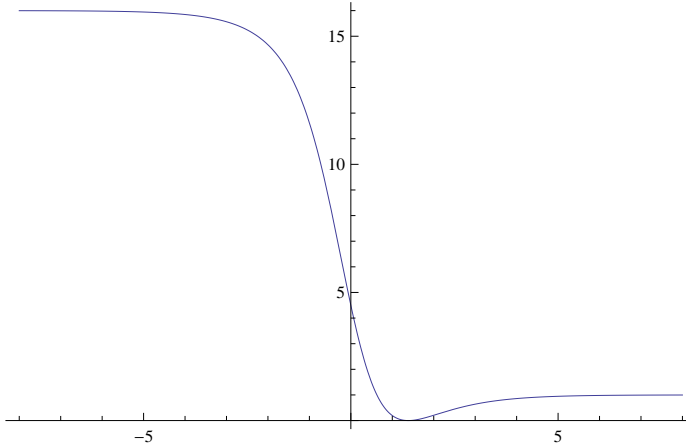
$$U_{\text{eff}}(\varphi) \simeq U_{(-)} \equiv \frac{f_1^2/f_2}{4\chi_2(1 + \epsilon f_1^2/f_2)} , \quad (27)$$

$$A(\varphi) \simeq A_{(-)} \equiv \frac{1 + \frac{1}{2}bf_1/f_2}{1 + \epsilon f_1^2/f_2} , \quad B(\varphi) \simeq B_{(-)} \equiv -\chi_2 \frac{b^2/4f_2 - \epsilon(1 + bf_1/f_2)}{1 + \epsilon f_1^2/f_2} . (28)$$

In the second flat region for large positive φ:

$$U_{\text{eff}}(\varphi) \simeq U_{(+)} \equiv \frac{M_1^2/M_2}{4\chi_2(1 + \epsilon M_1^2/M_2)} , \quad (29)$$

$$A(\varphi) \simeq A_{(+)} \equiv \frac{M_2}{M_2 + \epsilon M_1^2} , \quad B(\varphi) \simeq B_{(+)} \equiv \epsilon\chi_2 \frac{M_2}{M_2 + \epsilon M_1^2} . \quad (30)$$

Fig. 2 Shape of the effective scalar potential $U_{\text{eff}}(\varphi)$ (26) for $M_1 > 0$.

From the expression for $U_{\text{eff}}(\varphi)$ (26) and the figures 1 and 2 we see that now we have an explicit realization of quintessential inflation scenario. The flat regions (27)-(28) and (29)-(30) correspond to the evolution of the early and the late universe, respectively, provided we choose the ratio of the coupling constants in the original scalar potentials versus the ratio of the scale-symmetry breaking integration constants to obey:

$$\frac{f_1^2/f_2}{1 + \epsilon f_1^2/f_2} \gg \frac{M_1^2/M_2}{1 + \epsilon M_1^2/M_2} \,, \tag{31}$$

which makes the vacuum energy density of the early universe $U_{(-)}$ much bigger than that of the late universe $U_{(+)}$ (cf. (27), (29)). The inequality (31) is equivalent to the requirements:

$$\frac{f_1^2}{f_2} \gg \frac{M_1^2}{M_2} \quad , \quad |\epsilon| \frac{M_1^2}{M_2} \ll 1 \,. \tag{32}$$

In particular, if we choose the scales of the scale symmetry breaking integration constants $|M_1| \sim M_{EW}^4$ and $M_2 \sim M_{Pl}^4$, where M_{EW}, M_{Pl} are the electroweak and Plank scales, respectively, we are then naturally led to a very small vacuum energy density $U_{(+)} \sim M_1^2/M_2$ of the order $U_{(+)} \sim M_{EW}^8/M_{Pl}^4 \sim 10^{-120} M_{Pl}^4$, which is the right order of magnitude for the present epoche's vacuum energy density as already recognized in Ref.[26]. On the other hand, if we take the order of magnitude of the coupling constants in the effective potential $f_1 \sim f_2 \sim (10^{-2} M_{Pl})^4$, then together with the above choice of order of magnitudes for $M_{1,2}$ the inequalities (32) will be satisfied as well and the order of magnitude of the vacuum energy density of the early universe $U_{(-)}$ (27) becomes $U_{(-)} \sim f_1^2/f_2 \sim 10^{-8} M_{Pl}^4$ which conforms to the BICEP2 data [27] implying the energy scale of inflation of order $10^{-2} M_{Pl}$.

Let us recall that, since we are using units where $G_{\text{Newton}} = 1/16\pi$, in the present case $M_{Pl} = \sqrt{1/8\pi G_{\text{Newton}}} = \sqrt{2}$.

Before proceeding to the derivation of the non-singular "emergent universe" solution describing an initial phase of the universe evolution preceding the inflationary phase, let us briefly sketch how the present non-Riemannian-measure-

modified gravity-matter theory meets the conditions for the validity of the "slow-roll" approximation [5] when φ evolves on the flat region of the effective potential corresponding to the early universe (27)-(28).

To this end let us recall the standard Friedman-Lemaitre-Robertson-Walker space-time metric [25]:

$$ds^2 = -dt^2 + a^2(t)\Big[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\Big] \tag{33}$$

and the associated Friedman equations (recall the presently used units $G_{\text{Newton}} = 1/16\pi$):

$$\frac{\ddot{a}}{a} = -\frac{1}{12}(\rho + 3p) \quad, \quad H^2 + \frac{K}{a^2} = \frac{1}{6}\rho \quad, \quad H \equiv \frac{\dot{a}}{a}\,, \tag{34}$$

describing the universe' evolution. Here:

$$\rho = \frac{1}{2}A(\varphi)\,\dot{\varphi}^2 + \frac{3}{4}B(\varphi)\,\dot{\varphi}^4 + U_{\text{eff}}(\varphi)\,, \tag{35}$$

$$p = \frac{1}{2}A(\varphi)\,\dot{\varphi}^2 + \frac{1}{4}B(\varphi)\,\dot{\varphi}^4 - U_{\text{eff}}(\varphi) \tag{36}$$

are the energy density and pressure of the scalar field $\varphi = \varphi(t)$. Henceforth the dots indicate derivatives with respect to the time t.

Let us now consider the standard "slow-roll" parameters [6]:

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} \quad, \quad \eta \equiv -\frac{\ddot{\varphi}}{H\,\dot{\varphi}}\,, \tag{37}$$

where ε measures the ratio of the scalar field kinetic energy relative to its total energy density and η measures the ratio of the fields acceleration relative to the "friction" ($\sim 3H\,\dot{\varphi}$) term in the pertinent scalar field equations of motion:

$$\ddot{\varphi}\,(A + 3B\,\dot{\varphi}^2) + 3H\,\dot{\varphi}\,(A + B\,\dot{\varphi}^2) + U'_{\text{eff}} + \frac{1}{2}A'\,\dot{\varphi}^2 + \frac{3}{4}B'\,\dot{\varphi}^4 = 0\,, \tag{38}$$

with primes indicating derivatives w.r.t. φ.

In the slow-roll approximation one ignores the terms with $\ddot{\varphi}, \dot{\varphi}^2, \dot{\varphi}^3, \dot{\varphi}^4$ so that the φ-equation of motion (38) and the second Friedman Eq.(34) reduce to:

$$3AH\,\dot{\varphi} + U'_{\text{eff}} = 0 \quad, \quad H^2 = \frac{1}{6}U_{\text{eff}}\,. \tag{39}$$

The reason for ignoring the spatial curvature term K/a^2 in the second Eq.(39) is due to the fact that φ evolves on a flat region of U_{eff} and the Hubble parameter $H \equiv \dot{a}/a \simeq \text{const}$, so that $a(t)$ grows exponentially with time making K/a^2 very small. Consistency of the slow-roll approximation implies for the slow-roll parameters (37), taking into account (39), the following inequalities:

$$\varepsilon \simeq \frac{1}{A}\left(\frac{U'_{\text{eff}}}{U_{\text{eff}}}\right)^2 \ll 1 \quad, \quad \eta \simeq \frac{2}{A}\frac{U''_{\text{eff}}}{U_{\text{eff}}} - \varepsilon - \frac{2A'}{A^{3/2}}\sqrt{\varepsilon} \;\to\; \frac{2}{A}\frac{U''_{\text{eff}}}{U_{\text{eff}}} \ll 1\,. \tag{40}$$

Since now φ evolves on the flat region of U_{eff} for large negative values (27), the Lagrangian coefficient function $A(\varphi) \simeq A_{(-)}$ as in (28) and the gradient of the effective scalar potential is:

$$U'_{\text{eff}} \simeq -\frac{\alpha f_1 M_1 e^{\alpha\varphi}}{2\chi_2 f_2(1 + \epsilon f_1^2/f_2)^2}\,, \tag{41}$$

which yields for the slow-roll parameter ε (40):

$$\varepsilon \simeq \frac{4\alpha^2 M_1^2 e^{2\alpha\varphi}}{f_1^2(1+bf_1/2f_2)(1+\epsilon f_1^2/f_2)} \ll 1 \quad \text{for large negative } \varphi . \tag{42}$$

Similarly, for the second slow-roll parameter we have:

$$|\frac{2}{A}\frac{U''_{\text{eff}}}{U_{\text{eff}}}| \simeq \frac{4\alpha^2 |M_1| e^{\alpha\varphi}}{f_1(1+bf_1/2f_2)} \ll 1 \quad \text{for large negative } \varphi . \tag{43}$$

The value of φ at the end of the slow-roll regime φ_{end} is determined from the condition $\varepsilon \simeq 1$ which through (42) yields:

$$e^{-2\alpha\varphi_{\text{end}}} \simeq \frac{4\alpha^2 M_1^2}{f_1^2(1+bf_1/2f_2)(1+\epsilon f_1^2/f_2)} \tag{44}$$

The amount of inflation when φ evolves from some initial value φ_{in} to the end-point of slow-roll inflation φ_{end} is determined through the expression for the *e-foldings* N (see, *e.g.* second Ref.[2]):

$$N = \int_{\varphi_{\text{in}}}^{\varphi_{\text{end}}} H dt = \int_{\varphi_{\text{in}}}^{\varphi_{\text{end}}} \frac{H}{\dot{\varphi}} d\varphi \simeq - \int_{\varphi_{\text{in}}}^{\varphi_{\text{end}}} \frac{3H^2 A}{U'_{\text{eff}}} d\varphi \simeq - \int_{\varphi_{\text{in}}}^{\varphi_{\text{end}}} \frac{A U_{\text{eff}}}{2U'_{\text{eff}}} d\varphi , \tag{45}$$

where Eqs.(39) are used. Substituting (27), (28) and (41) into (45) yields an expression for N which together with (44) allows for the determination of φ_{in}:

$$N \simeq \frac{f_1(1+bf_1/f_2)}{4\alpha^2 M_1}\left(e^{-\alpha\varphi_{\text{in}}} - e^{-\alpha\varphi_{\text{end}}}\right) . \tag{46}$$

4 Non-Singular Emergent Universe Solution

We will now show that under appropriate restrictions on the parameters there exist an epoch preceding the inflationary phase. Namely, we derive an explicit cosmological solution of the Einstein-frame system with effective scalar field Lagrangian (23)-(26) describing a non-singular "emergent universe" [14] when the scalar field evolves on the first flat region for large negative φ (27). For previous studies of "emergent universe" scenarios within the context of the less general modified-measure gravity-matter theories with one non-Riemannian and one standard Riemannian integration measures, see Ref.[20].

Emergent universe is defined through the standard Friedman-Lemaitre-Robertson-Walker space-time metric (33) as a solution of (34) subject to the condition on the Hubble parameter H:

$$H = 0 \quad \rightarrow \quad a(t) = a_0 = \text{const}, \quad \rho + 3p = 0 \quad , \quad \frac{K}{a_0^2} = \frac{1}{6}\rho \; (= \text{const}), \tag{47}$$

with ρ and p as in (35)-(36):

The emergent universe condition (47) implies that the φ-velocity $\dot{\varphi} \equiv \dot{\varphi}_0$ is time-independent and satisfies the bi-quadratic algebraic equation:

$$\frac{3}{2} B_{(-)} \, \dot{\varphi}_0^4 + 2A_{(-)} \, \dot{\varphi}_0^2 - 2U_{(-)} = 0 \tag{48}$$

(with notations as in (27)-(28)), whose solution read:

$$\dot{\varphi}_0^2 = -\frac{2}{3B_{(-)}}\left[A_{(-)} \mp \sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}\right] . \tag{49}$$

Let us note that according to (28) $B_{(-)} < 0$ for a wide range of the parameters, in particular, within the allowed interval of stability (see (56) below). We also observe that under the emergent universe condition (47), and since now $\dot{\varphi}$ is time-independent, the φ-equations of motion (38) are identically satisfied.

To analyze stability of the present emergent universe solution:

$$a_0^2 = \frac{6k}{\rho_0} \quad , \quad \rho_0 = \frac{1}{2}A_{(-)}\,\dot{\varphi}_0^2 + \frac{3}{4}B_{(-)}\,\dot{\varphi}_0^4 + U_{(-)} , \tag{50}$$

with $\dot{\varphi}_0^2$ as in (49), we perturb Friedman Eqs.(34) and the expressions for ρ, p (35)-(36) w.r.t. $a(t) = a_0 + \delta a(t)$ and $\dot{\varphi}(t) = \dot{\varphi}_0 + \delta\dot{\varphi}(t)$, but keep the effective potential on the flat region $U_{\text{eff}} = U_{(-)}$:

$$\frac{\delta\ddot{a}}{a_0} + \frac{1}{12}(\delta\rho + 3\delta p) \quad , \quad \delta\rho = -\frac{2\rho_0}{a_0}\delta a , \tag{51}$$

$$\delta\rho = \left(A_{(-)}\,\dot{\varphi}_0 + 3B_{(-)}\,\dot{\varphi}_0^3\right)\delta\dot{\varphi} = -\frac{2\rho_0}{a_0}\delta a \,\, , \,\, \delta p = \left(A_{(-)}\,\dot{\varphi}_0 + B_{(-)}\,\dot{\varphi}_0^3\right)\delta\dot{\varphi} . \tag{52}$$

From the first Eq.(52) expressing $\delta\dot{\varphi}$ as function of δa and substituting into the first Eq.(51) we get a harmonic oscillator type equation for δa:

$$\delta\ddot{a} + \omega^2\delta a = 0 \quad , \quad \omega^2 \equiv \frac{2}{3}\rho_0\frac{\pm\sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}}{A \mp 2\sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}} , \tag{53}$$

where:

$$\rho_0 \equiv \frac{1}{2}\dot{\varphi}_0^2\left[A_{(-)} + 2\sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}\right] , \tag{54}$$

with $\dot{\varphi}_0^2$ from (49). Thus, for existence and stability of the emergent universe solution we have to choose the upper signs in (49), (53) and we need the conditions:

$$A_{(-)}^2 + 3B_{(-)}U_{(-)} > 0 \quad , \quad A_{(-)} - 2\sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}] > 0 . \tag{55}$$

The latter yield the following constraint on the coupling parameters:

$$\max\left\{-2, -8(1 + 3\epsilon f_1^2/f_2)\left[1 - \sqrt{1 - \frac{1}{4(1 + 3\epsilon f_1^2/f_2)}}\right]\right\} < b\frac{f_1}{f_2} < -1 , \tag{56}$$

in particular, implying that $b < 0$. The latter means that both terms in the original matter Lagrangian $L^{(2)}$ (4) appearing multiplied by the second non-Riemannian integration measure density Φ_2 (2) must be taken with "wrong" signs in order to have a consistent physical Einstein-frame theory (23)-(25) possessing a non-singular emergent universe solution.

For $\epsilon > 0$, since the ratio $\frac{f_1^2}{f_2}$ proportional to the height of the first flat region of the effective scalar potential, $i.e.$, the vacuum energy density in the early universe, must be large (cf. (31)), we find that the lower end of the interval in (56) is very close to the upper end, $i.e.$, $b\frac{f_1}{f_2} \simeq -1$.

For a recent semiclassical analysis of quantum (in)stability of oscillating emergent universes we refer to [28,29].

5 Discussion

In the present paper we have constructed a new kind of gravity-matter theory defined in terms of two different non-Riemannian volume-forms (generally covariant integration measure densities) on the space-time manifold, where the Einstein-Hilbert term R, its square R^2, the kinetic and the potential terms in the pertinent cosmological scalar field (a "dilaton") couple to each of the non-Riemannian integration measures in a manifestly globally Weyl-scale invariant form. The principal results are as follows:

– Dynamical spontaneous symmetry breaking of the global Weyl-scale invariance.
– In the physical Einstein frame we obtain an effective scalar field potential with *two flat regions* – one corresponding to the early universe evolution and a second one for the present slowly accelerating phase of the universe.
– The flat region of the effective scalar potential appropriate for describing the early universe allows for the existence of a *non-singular "emergent"* type beginning of the universe' evolution. This "emergent" phase is followed by the inflationary phase, which in turn is followed by a period, where the scalar field drops from its high energy density state to the present slowly accelerating phase of the universe.

The flatness of the effective scalar potential in the high energy density region makes the slow rolling inflation regime possible.

The presence of the emergent universe' phase preceding the inflationary phase has observable consequences for the low CMB multipoles as has been recently shown in Ref.[30]. For a systematic unified analysis of all stages of the cosmological scenario developed here (emergent universe, transition from emergent universe to inflation – period of "super-inflation", "slow roll" regime, *etc.*) it will be instructive to employ the methods of dynamical systems' evolution [31].

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